

Linking halo mass to galaxy luminosity

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ABSTRACT

In this paper we present a new, essentially empirical, model for the relation between the mass of a dark matter halo/subhalo and the luminosity of a galaxy hosted in it. To estimate this, we replace the assumption of linearity between light and mass fluctuations with the assumption of monotonicity between galaxy light and halo or subhalo mass. We are enabled to proceed with this less restrictive ansatz by the availability of new, very high resolution dark matter simulations and more detailed and comprehensive global galactic luminosity functions.

We find that the relation between halo/subhalo mass and hosted galaxy luminosity, is fairly well fit by a double power law. That between halo mass and group luminosity has a shallower slope for an intermediate mass region, and is fairly well fit by a two branch function, with both branches double power laws. Both relations asymptote to $L \propto M^4$ at low M , while at high mass the former follows $L \propto M^{0.28}$ and the latter $L \propto M^{0.9}$.

In addition to the mass-luminosity relation, we also derive results for the occupation number, luminosity function of cluster galaxies, group luminosity function and multiplicity function. Then, using a prescription for the mass function of haloes in under/overdense regions and some further assumptions on the form of the mass density distribution function, we further derive results for biasing between mass and light and mass and galaxy number, light distribution function and the void probability distribution.

Our results for the most part seem to match well with observations and previous expectations. We feel this is a potentially powerful way of modelling the relation between halo mass and galaxy luminosity, since the main inputs are readily testable against dark matter simulation results and galaxy surveys, and the outputs are free from the uncertainties of physically modelling galaxy formation.

Key words: galaxies: haloes – cosmology: theory – dark matter – large-scale structure of the universe

1 INTRODUCTION

In recent years, N-body numerical simulations have given us a good understanding of dark matter structure for standard cosmological scenarios, while large scale observational surveys have done the same for the distribution of galaxies. In this way, we are now developing a good picture of how mass and luminosity are distributed in the universe. However, it is still not well known how to connect the two pictures. While it is well established that dark matter haloes are the hosts of the observed galaxies, it is still poorly understood how the former are related to the latter. Further, the picture is complicated by the fact that what is usually taken as a halo in simulations would often host multiple galaxies, especially for higher masses. To analyse the issue fully, it is necessary to

look at the halo substructure, since each subhalo can host a galaxy. Establishing such a link between halo mass and galaxy luminosity would be important because, first of all, it would allow us to have a direct connection between theory and observation, dark matter haloes and galaxies. Further, it could also shed some light into the theory of galaxy formation.

There are several ways in which this problem can be studied. The more direct ones involve either numerical simulations including gas dynamics (White, Hernquist, & Springel 2001; Yoshikawa et al. 2001; Pearce et al. 2001; Nagamine et al. 2001; Berlind et al. 2003), or semi-analytical models of galaxy formation (Kauffmann, Nusser, & Steinmetz 1997; Governato et al. 1998; Kauffmann et al. 1999a,b; Benson et al. 2000a,b; Sheth & Diaferio 2001; Somerville et al. 2001; Wechsler et al. 2001; Benson et al. 2003a; Berlind et al.

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2003), but, while they explicitly give the properties of galaxies located in a given halo, they have the added difficulty that many of the mechanisms involved in galaxy formation are poorly understood, and difficult to compute. Their complexity could also mask any fundamental relations that might be present between halo and galaxy properties.

More indirect approaches have also been studied. The halo occupation distribution (HOD) model (Seljak 2000; Benson 2001; Bullock, Wechsler, & Somerville 2002; Zheng et al. 2002; Berlind & Weinberg 2002; Berlind et al. 2003; Magliocchetti & Porciani 2003) is based on the probability $P(N|M)$ that a halo of mass M is host to N galaxies. By specifying the $P(N|M)$ function, along with some form for the distribution of dark matter and galaxies within each halo, it is then possible to relate different statistical indicators of the dark matter and galaxy distributions, such as correlation functions, to each other. This fully specifies the bias between the galaxy and the underlying matter distributions. A recent paper by Kravtsov et al. (2003), has done a detailed study of results from simulations and related them to the HOD model, and has concluded that the form of $P_s(N_s|\mu)$ for the subhaloes is approximately universal, where μ is the subhalo mass scaled to an appropriate minimum mass. This paper also give results for the relation between galaxy absolute magnitude and halo circular velocity. Other work (van den Bosch, Yang & Mo 2003; Yang, Mo & van den Bosch 2003; Mo et al. 2003) has taken this approach one step further by studying not only the number of galaxies associated with each halo, but also their luminosity, by building the conditional luminosity function, $\Phi(L|M)dL$. This gives the number of galaxies with luminosities in the range $L \pm dL/2$ contained in a halo of mass M . While this work directly relates the halo mass to the galaxy luminosity, it does so only to the average values, lacking the full statistical treatment which is analysed in the HOD models.

Other authors have used a slightly different method. Instead of trying to specify the number of galaxies in each halo, they treat the halo as a whole and identify it with a galaxy group. Then, by comparing the group luminosity function with the halo mass function, they obtain the luminosity associated with each halo (Peacock & Smith 2000; Marinoni & Hudson 2002), and also develop ways to estimate the number of galaxies hosted in a halo, thus coming back partly to the $P(N|M)$ estimate of the HOD models.

In the present paper, we follow a new and conceptually clear approach based on one simplified and testable hypothesis: there is a one to one, monotonic correspondence between halo/subhalo mass and resident galaxy luminosity. We might call this the empirical (rather than the semi-analytical) approach because there is no attempt to physically model the galaxy formation process. Instead we take from observations the galaxy luminosity distribution and match it with the theoretical halo/subhalo distribution. This has the additional advantage of naturally giving a lower mass threshold for haloes that host luminous galaxies, as the luminosity decreases sharply with mass for less massive haloes. It also implicitly gives rise to galaxy systems, if one identifies a system of a massive halo and its subhaloes with the central galaxy and its satellites in groups and clusters. We find that the single assumption is very powerful and allows us to compute, and compare to observations many quantities

from bias to the void distribution function to the spatial correlation function.

This paper is organized as follows: in section 2 we present our model for the subhalo mass distribution in a parent halo, and build the global subhalo mass function. In section 3, we derive the relation between mass and luminosity, as well as some other functions such as the luminosity function of cluster galaxies, the group luminosity function and the multiplicity function. In section 4 we study how to apply the relation we obtain to get the light density and the number density of galaxies as a function of mass density, and also obtain results for the distribution function of light density and the void probability function. Finally, we conclude in section 5.

Throughout we have used a concordance cosmological model, with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$ and $\sigma_8 = 0.9$ (Bahcall et al. 1999; Spergel et al. 2003).

2 SUBHALO DISTRIBUTION

The first step in building our model is to specify the mass distribution of subhaloes for a given parent halo. We adopt the following function:

$$N(m|M)dm = A \left(\frac{m}{x\beta M} \right)^{-\alpha} \exp \left(- \frac{m}{x\beta M} \right) \frac{dm}{x\beta M}, \quad (1)$$

which gives the number of subhaloes with masses in the range m to $m + dm$, for a parent halo of mass M . The normalisation A is such that the total mass in these subhaloes, $\int_0^\infty mN(m|M)dm$, is a fraction of the parent halo mass, $x\gamma M$ (where the factor x accounts for the added mass of the original, unstripped, subhaloes). With this definition, we can write A as

$$A = \frac{\gamma}{\beta\Gamma(2-\alpha)}. \quad (2)$$

This expression is motivated by recent analysis of high resolution dark matter simulations of Weller, Ostriker & Bode (2004), and we use for the parameters the values $\alpha = 1.91$, $\beta = 0.39$ and $\gamma = 0.18$. Its results are similar to those obtained by the simulations of De Lucia et al. (2004), who find a power law fit to the subhalo mass function with a slope close to -2 (that is, in terms of equation 1, $\alpha = 2$). However, they also find that if they only include the lowest mass bins where statistical errors are smallest, this slope is reduced to values around -1.9, very similar to what we use here. It further agrees well with the cumulative mass function derived analytically by Oguri & Lee (2004) (see section below on the occupation numbers for further discussion), and it is also similar to the power law form for the subhalo number given in Sheth & Jain (2003), only instead of having a sharp cutoff at the parent halo mass, we introduce a smooth exponential cutoff from a mass βM . Since it is a Schechter function, it is also similar to the halo mass function, and the slope α is close to the expected value of the slope of the halo mass function.

It gives a total mass in subhaloes of 18%, close to but slightly higher than the values of $\approx 10\%$ obtained in different studies (see, for example, Tormen, Diaferio & Syer 1998; Ghigna et al. 2000). De Lucia et al. (2004) derive a

lower value of around 6% at radius r_{200} (where the average overdensity is 200 times the critical), though they also find that this fraction can be as high as 10-15% in some cases. However, r_{200} is smaller than the virial radii typically measured from the simulation results by Weller, Ostriker & Bode (2004), which helps to explain the difference.

It is worth noting that, like previous results from simulations and analytical modelling (e.g., Moore et al. 1999; Kravtsov et al. 2003; De Lucia et al. 2004; Oguri & Lee 2004), the shape of the subhalo mass function given by equation 1 is independent of parent halo mass. This does not mean, however, that one should automatically expect the same to be true of satellite galaxy distribution in galaxies like the Milky Way and clusters. In fact, as can be seen from our results further below, the mass luminosity relation has very different behaviour depending on the mass of the host halo/subhalo; this leads to parent halos of very different masses (such as a cluster and a galaxy) having very different satellite luminosity distributions, even though the scaled mass functions of their subhaloes are identical.

A very important point when analysing this expression is to note exactly what mass is being accounted by m . In fact, the distribution is valid for the present mass of the subhalo satellites, obtained when $x = 1$, which is measured after the tidal stripping of the outer parts of the subhalo in the halo potential well. However, to use this function to build the host distribution and to subsequently compare it to the galaxy luminosity function, it is necessary to use the original, unstripped mass of the subhalo, since only then can we establish a monotonic correspondence between host halo mass and galaxy luminosity. The factor x put into the expression takes this mass loss into account, since we can then treat the mass m in equation (1) as the original mass, with the stripped mass being then m/x with $x > 1$. Obviously, the actual factor for a given subhalo will be highly variable, but for reasons of simplicity and also because it is not well known (see e.g. Hayashi et al. 2003 for a treatment of the profile of stripped subhaloes), we will be treating this as an average applicable to all subhaloes, and using $x = 3$ in the present work. It should be noted that, since we are taking the total subhalo mass to be 18% of the parent halo mass, this factor cannot be more than 5, assuming that the majority of the mass in the parent halo was built up by stripping of the subhaloes. A recent paper by Kravtsov et al. (2004) includes a comparison between maximum and present mass of subhaloes in the dwarf mass range. Their results (see their figure 4) seem to support that the average change in mass does not depend on the mass of the subhalo, and an average mass stripping factor of 3 as we adopt here seems a fair agreement.

All plots and further expressions (where we will drop the factor x) presented refer to this original, unstripped mass; to invert our approximation is simply a case of dividing the subhalo mass by a factor of 3 to represent the actual stripped mass which would be measured in a simulation. Ideally, we should use instead a distribution for the maximum circular velocity of the subhaloes. Even though this quantity also shows a large scatter between the maximum value and that at present after stripping (Kravtsov et al. 2004), its relative change should be smaller than what would be expected for a given change in mass. This should be the case especially

for subhaloes massive enough to host galaxies, which would have only their outer parts stripped off, and where therefore the inner parts which determine the peak circular velocity are left relatively undisturbed. This would mean that a calculation based on the cumulative number function like we use here would incur a smaller error, even with a large scatter, and in later work we will transform to that variable.

Using the expression for the subhalo number (1) together with the halo mass function, $n_h(M)$, it is then possible to build the global subhalo mass function, that is, the number density of subhaloes in a given mass range, by

$$n_{sh}(m) = \int_0^\infty N(m|M)n_h(M)dM. \quad (3)$$

If we assume that the halo mass function has a Schechter form,

$$n_h(M)dM = C\left(\frac{M}{M_*}\right)^{-a} \exp\left(-\frac{M}{M_*}\right) \frac{dM}{M_*}, \quad (4)$$

it is possible to write an analytical expression for it:

$$n_{sh}(m) = \frac{2C\gamma}{M_*\beta^2\Gamma[2-\alpha]}\left(\frac{m}{\beta M_*}\right)^{-\frac{a+\alpha}{2}} K_0\left(2\sqrt{\frac{m}{\beta M_*}}\right), \quad (5)$$

where K stands for the modified Bessel function.

With more general mass functions, the calculation has to be done numerically. In the present work, we use the more accurate Sheth Tormen mass function (Sheth & Tormen 1999),

$$n_h(M)dM = A\left(1 + \frac{1}{\nu^{2q}}\right) \sqrt{\frac{2}{\pi}} \frac{\rho_m}{M} \frac{d\nu}{dM} \exp\left(-\frac{\nu^2}{2}\right) dM, \quad (6)$$

with $\nu = \sqrt{a} \frac{\delta_c}{D(z)\sigma(M)}$, $a = 0.707$, $A \approx 0.322$ and $q = 0.3$; as usual, $\sigma(M)$ is the variance on the mass scale M , $D(z)$ is the growth factor, and δ_c is the linear threshold for spherical collapse, which in the case of a flat universe is $\delta_c = 1.686$. The result for the global subhalo mass function is shown in figure 1, where the Sheth Tormen mass function is also shown.

2.1 Occupation number

With the expression for the subhalo number, equation (1), it is also possible to calculate the halo occupation number, that is, the number of subhaloes in a halo of mass M :

$$N_s = \frac{\gamma}{\beta\Gamma(2-\alpha)}\Gamma(1-\alpha, \frac{m_{min}}{\beta M}), \quad (7)$$

Because the integral diverges, it is necessary to set a minimum threshold for the subhaloes. This can be set as the minimum mass for a halo to host a galaxy, in which case this occupation number corresponds to the number of galaxies contained in a given halo. This is one of the basic ingredients of HOD models, where it is taken as the average number of galaxies in a halo. Figure 2 shows the occupation number as a function of the parent halo mass, in units of the minimum mass considered. The most prominent feature is the cutoff for $M/M_{min} \lesssim 5$; this is inherent to our model, since we consider the total mass in subhaloes to be only 18% of the halo mass, and we also have a cutoff in the number of subhaloes, given by the parameter β . At the high mass end, the function is well fit by a power law, with $N_s \propto M^{0.91}$

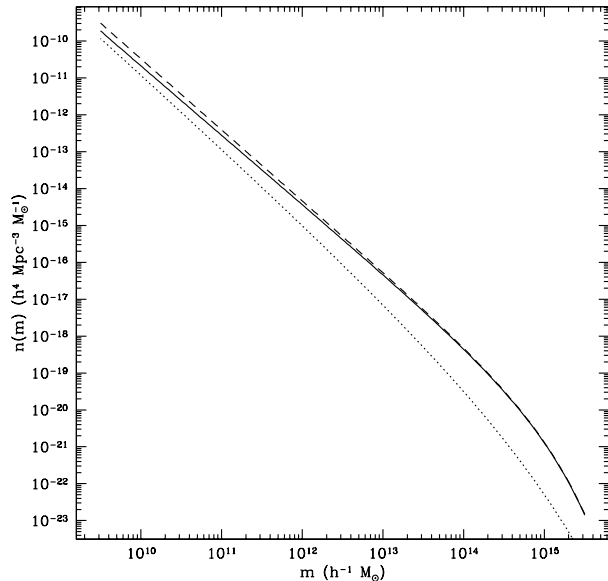


Figure 1. Mass function for haloes (solid line) and subhaloes (dotted line). The dashed line is the sum of the two. The parent halo mass function is the one proposed by Sheth Tormen (equation 6), while the subhalo one is the one obtained using our model for the subhalo number distribution with parent halo mass.

for $(M/M_{min}) \gtrsim 1000$, which is similar to what is predicted by most HOD models (see e.g., van den Bosch, Yang & Mo 2003; Magliocchetti & Porciani 2003; Kravtsov et al. 2003).

This behaviour is also analogous to that found in the analytically derived halo mass function of Oguri & Lee (2004), who also find a slope of roughly 0.9 at high (M/M_{min}) , but a steeper value close to 1 in the lower range, similar to our results. The values they obtain are also similar to ours, with one caveat: they correspond to those we would get if we took the values for the stripped mass of the subhaloes, instead of what is plotted in figure 2; this would correspond roughly to dividing the numbers we get by a factor of 3. Our result is also similar to what has been observed for the case of galaxies in a cluster, with Kochanek et al. (2003) obtaining a relation for the number of galaxies with luminosity greater than L_* , $N_g(L > L_*) \propto M_h^{1.1}$, where all quantities are normalised to the usual mass overdensity of 200. If we assume that their L_* galaxy would have a mass of around $10^{12} h^{-1} M_\odot$ (see below for the results we obtain), we also obtain approximately the same number of galaxies in a $10^{15} h^{-1} M_\odot$ parent halo. The overall shape of our occupation number is also similar to the results obtained by Kravtsov et al. (2003) in their simulations, although in their case the cutoff is less pronounced, and in fact is only noticeable for $M/M_{min} \approx 1$. As referred, however, our cutoff around $M/M_{min} \approx 5$ is in fact a feature of our model, and something similar should be present as long as a cutoff in subhalo mass is introduced and there is an upper limit to the total mass in subhaloes. The normalisation of the curves is also similar, with a value of $N_s \approx 30$ for $M/M_{min} \approx 1000$. It should however be cautioned that the way in which the subhalo mass is accounted may not be exactly the same in both cases (see discussion above).

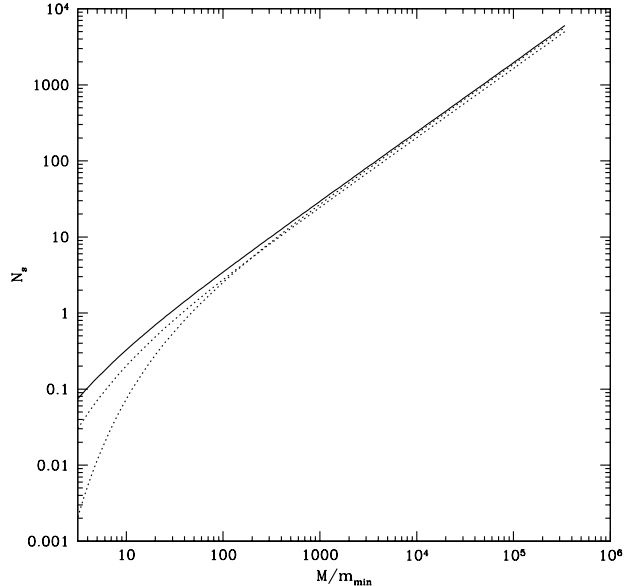


Figure 2. Number of subhaloes in a halo of mass M (equation 7, solid line). The halo mass is scaled to the minimum mass considered for the subhaloes. The solid line shows the results for the model used in this paper ($\beta = 0.39$). The two dotted lines show the result of changing the subhalo cutoff (respectively for $\beta = 0.2$ and $\beta = 0.05$).

3 MASS LUMINOSITY RELATION

Once we have the total mass function for haloes and their subhaloes, it is then possible to compare it to the galaxy luminosity function to obtain a mass luminosity relation. As noted, a few prior assumptions go into this: first, that the galaxies are hosted individually in the haloes or subhaloes, and that each contains a single one (or, in the case of the parent haloes, they have one in their centre, plus the ones in their subhaloes); and second, that the luminosity is a monotonic function of the halo mass. The first assumption is supported by previous studies of the group mass function. For example, Martínez et al. (2002) obtain a group mass function from the 2dF catalogue which matches well with theoretical mass functions like the Sheth Tormen one, equation (6), indicating that each of their identified groups corresponds to a halo. As for the second, Neyrinck, Hamilton, & Gnedin (2004) have shown that subhaloes identified in a set of simulations have a correlation function and power spectrum that matches the galaxies in the PSCz survey, which shows it is possible to understand the spatial distribution of galaxies by identifying galaxies brighter than a given luminosity with haloes larger than a certain mass. This is further justified by studies which show that the only halo property dependent on the large scale environment is the mass distribution (e.g., Lemson & Kauffmann 1999), which coupled with the current understanding of galaxy formation theories should imply that we have captured most of the environmental dependence of the galaxy luminosity.

We take the galaxy luminosity function to have the usual Schechter form:

$$\phi(L)dL = \phi_* \left(\frac{L}{L_*} \right)^\alpha \exp \left(- \frac{L}{L_*} \right) \frac{dL}{L_*}. \quad (8)$$

The values of the different parameters are taken from the b_J band 2dF galaxy luminosity function (Norberg et al. 2002), with $\Phi^* = 1.61 \times 10^{-2} h^3 \text{Mpc}^{-3}$, $M_{b_J}^* - 5 \log_{10} h = -19.66$ and $\alpha = -1.21$. Although this fit was determined for the magnitude range $-16.5 > M_{b_J} - 5 \log h > -22$, here we will be extrapolating its result to higher or lower luminosities as necessary. The mass luminosity relation is then calculated by setting the luminosity L of a galaxy hosted in a halo of mass M to be such that the number of galaxies with luminosity greater than L equals the number of haloes plus subhaloes with mass greater than M :

$$\int_L^\infty \phi(L)dL = \int_M^\infty [n_h(M) + n_{sh}(M)]dM. \quad (9)$$

It should be noted that, because the values for the subhalo mass function are generally lower than those of the halo one, the subhaloes make only a small contribution to this expression, only being important at lower mass values. This guarantees that a possible second generation of subhaloes (subhaloes of the subhaloes considered here) would not influence this result much, since their mass function would have values that much lower than the original halo mass function, and then only at the lowest masses considered.

It is also possible to obtain the group luminosity associated with each halo. In order to do this, we first assume that a galaxy system (which would be either an isolated galaxy, a group or a cluster, depending on the mass of the halo containing it) can be represented by a halo and its associated subhaloes. The group luminosity for a halo of mass M is then simply

$$L_g(M) = L(M) + \int_0^\infty L(m)N(m|M)dm. \quad (10)$$

There is an important point that should be considered when analysing this expression, and that is the possibility of putting a lower limit in the integral, since there may be a minimum mass for a halo to be host to a luminous galaxy. However, as can be seen in figure (3), which shows the results for the luminosity and group luminosity associated with a halo, the calculated luminosity decreases sharply with mass at the low end, becoming negligible for haloes with masses lower than approximately $10^{9.5} h^{-1} \text{M}_\odot$. Therefore, instead of putting in a cutoff, the value of which is not clear from what is known of galaxy formation, we simply use the natural drop off in the derived relation.

This lower limit seems interestingly to be roughly the mass which is usually considered for the host haloes of dwarf galaxies (for example, Thoul & Weinberg (1996) give a lower limit of $v_c = 30 \text{km/s}$ to the maximum circular velocity of halo capable of hosting a dwarf galaxy; see also Stoehr et al. (2002); Hayashi et al. (2003) for their simulation results on the halo hosts of dwarf galaxies, where they give similar values for the less massive of them), but it is quite low when compared with the values used in some studies (for example, in the HOD model in Berlind & Weinberg (2002)), although this may be related to the fact that most dark matter simulations used in these studies do not reach such low masses. Another important aspect that can be seen in the figure is that the group luminosity only starts to depart from the halo luminosity for masses above $10^{12} h^{-1} \text{M}_\odot$. This is to be

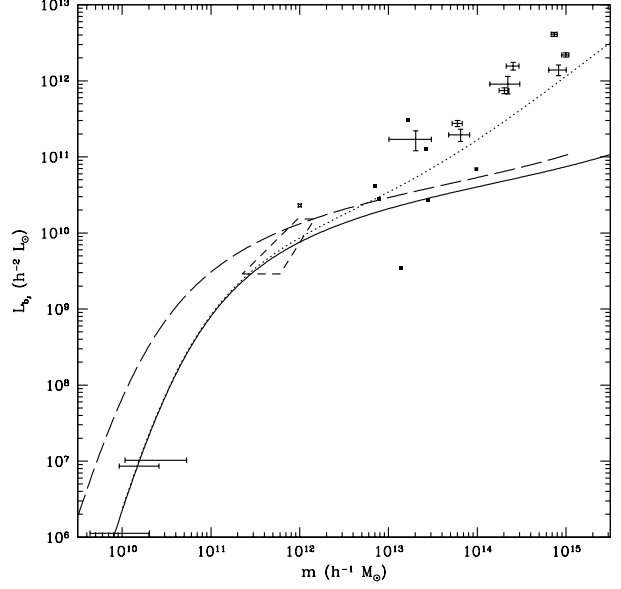


Figure 3. Relation between galaxy luminosity and the original mass of the dark matter halo which hosts it. The solid line is for each individual halo or subhalo, the dotted line shows the total group luminosity of the halo plus subhaloes system, in which case the x-axis m refers to the parent halo mass only. Also shown as a dashed line is the result when the mass is taken to be the actual stripped mass of the subhaloes (which is obtained by dividing the original mass by a factor of 3). The different points are estimates from different observational studies (see discussion in text for references).

expected given the estimated mass of the host haloes of rich groups and clusters. This difference is the contribution of the subhaloes, and it is clearly seen that, although their numbers may not be very significant to the total mass function, they are very important for the luminosity of high mass haloes, and therefore to the luminosity function. Due to their small numbers when compared with subhaloes, it is to be expected that subsubhaloes make only a small contribution to this group luminosity, and that this would only be noticeable for the most massive parent haloes, where the subsubhaloes can be massive enough to be luminous. Noticeably, the group luminosity has a middle region of intermediate mass with a shallower slope, and then an upturn for high mass, which was also present in similar studies which used the group luminosity function (e.g., Peacock & Smith 2000), while the single halo luminosity has a much shallower high end slope.

We find that the mass luminosity relation we obtain can be fairly well fit by a double power law of the type:

$$L = A \frac{(m/m')^b}{(c + (m/m')^{dk})^{1/k}}. \quad (11)$$

For the luminosity of individual galaxies (the solid line in figure 3), the parameters are $A = 5.7 \times 10^9$, $m' = 10^{11}$, $b = 4$, $c = 0.57$, $d = 3.72$ and $k = 0.23$. Therefore, at the low mass end, we have $L \propto M^4$. In fact, if we go to very low luminosities, the slope will actually be slightly steeper; nonetheless, the region of interest essentially begins for luminosities above a few times $10^5 h^{-2} L_\odot$ (which corresponds to the fainter dwarf galaxies; however, note that such low

luminosities are outside the range of the Schechter fit to the 2dF luminosity function, so this is based on an extrapolation of the 2dF results to this luminosity range), and for these the slope is approximately 4. Such a slope is actually what you would expect from a straightforward comparison between the halo mass function (assuming a slope of -1.8 at the low mass end) and the galaxy luminosity function. Since at the low mass end the subhaloes give an important contribution to the total number of hosts (see figure 1), this is most likely a coincidence, arising from the fact that the total host distribution (haloes+subhaloes) has a slope similar to the halo mass function. At the high mass end, we essentially obtain the relation between the halo mass and the luminosity of the brightest cluster galaxy, which has a much shallower slope ($L \propto M^{0.28}$). This is most likely due to the fact that, by construction, the mass term refers to the mass of the entire halo hosting the cluster, and not just to the mass in the region of the galaxy itself.

The halo mass / group luminosity relation is not really well fit by a double power law, since it has a third region for middle values of mass with a shallower slope than either of the asymptotic values. However, we find that it is possible to describe it as two different double power law branches, which provide a fair fit to the results. Thus, for $m < 10^{12} h^{-1} M_{\odot}$, the group luminosity is essentially the same as the luminosity of the parent halo (as these haloes do not have subhaloes massive enough to be luminous), so we fit it also with equation 11, and use the same parameters as before. For parent halo masses higher than $10^{12} h^{-1} M_{\odot}$, we find that a simple double power law of the form $4.8 \times 10^{10} (m_1^{0.9} + 0.6 m_1^{0.4})$, with $m_1 = m / (3.5 \times 10^{13} h^{-1} M_{\odot})$ is a good fit. Such a behaviour, of a curve with a relatively flat slope in the intermediate mass range and steeper slopes at both the low and high mass ends is similar to what was observed by Peacock & Smith (2000), who compared the AGS group luminosity function with the halo mass function, although the actual shape of the curve is somewhat different from what we find here.

At the high mass end, the cluster luminosity is almost directly proportional to halo mass (in fact, $L \propto M^{0.9}$). This means that the resulting mass to light ratio will be almost constant, rising only very slowly with halo mass, which matches well with previous results for the mass to light ratio of clusters (e.g., Bahcall et al. 2000; Kochanek et al. 2003). However, the values we obtain for the group luminosity seem to be smaller than the observational results. Further, the value derived by Fukugita, Hogan & Peebles (1998) for the cluster blue mass to light ratio is $450 \pm 100 h(M/L)_{\odot}$, which is roughly consistent with the value we obtain around $10^{13} - 10^{14} h^{-1} M_{\odot}$, but smaller than what we get at $10^{15} h^{-1} M_{\odot}$. Since the mass luminosity relation for a single halo is dominated at the high end by the parent haloes, it seems unlikely that this result could be much modified in the scope of our model, since both the halo mass function and the luminosity function are well known. This then means that the problem would lie in the subhalo distribution for these massive parent haloes; it would be necessary either to have more of them, or else for them to be more luminous. The two are in fact related, since an increase in the number of subhaloes causes an increase in the total number of hosts, and thus a decrease in the luminosity of a halo (especially in the low mass range, where subhalo number is more significant). However, our results for the occupation number and the luminosity of lower

mass haloes seem to be in good agreement with observations and prior theoretical models, which leads us to believe that the most likely cause of this result is the simplistic way in which we treated subhalo mass stripping.

We can also compare our results with those in van den Bosch, Yang & Mo (2003), who fit the mass to light ratio in different models they study to a double power law. In general, their results are in fair agreement with what we obtain. At the low mass end, the results are quite similar, with these authors obtaining a minimum of the mass to light ratio at a slightly lower mass. At the high mass end, they obtain a steeper mass to light ratio as a function of mass, although this is due to the flattening off we find in our results, since in the intermediate range $10^{13} - 10^{14} h^{-1} M_{\odot}$ we obtain higher mass to light ratios and a steeper relation. This discrepancy is most likely due to the factor that these authors are fitting the mass to light ratio to a double power law, which as was discussed above does not provide a good fit to our results. In fact, when they adopt a different model with a fixed mass to light ratio at high mass, their results agree with ours slightly better in this intermediate region.

Overall, our results seem a fairly good match to estimates of mass taken from a range of observational results across the entire mass range. Shown are points for the three most luminous of the dwarf spheroidals in the local group, where the luminosity was taken from the review by Mateo (1998), and the mass was estimated from the results of Hayashi et al. (2003) (see the results in their figure 13 for bounds to the peak circular velocity function of unstripped NFW haloes estimated to be possible hosts of the local group dwarf spheroidals), by assuming that the relation between mass and luminosity is monotonic. The dashed box represents the relation obtained from the weak lensing study of Hoekstra, Yee, & Gladders (2004), for galaxies around L^* (where we have taken the values applicable to a NFW halo). The results for poor groups are taken from Ramella et al. 2002, while those for clusters are from Girardi et al. 2000, where the bounds come from the two different methods the authors use for estimating fore/background corrections. Also included is a point for the Milky Way (Cox 2000).

3.1 Luminosity function of cluster galaxies

The major difficulty in obtaining a luminosity function for galaxies in clusters lies in distinguishing what haloes should be treated as rich clusters in our model, and which are simply groups. We adopt the Abell definition for rich cluster, namely that it must have upwards of 30 objects brighter than $m_3 + 2^m$, where m_3 is the magnitude of the third brightest galaxy in the cluster. Using our derived mass luminosity relation, it is possible to transform this magnitude threshold into a mass one. In our model, this would correspond to the mass of the second most massive subhalo (since we consider the brightest galaxy to be in the parent halo). In order to calculate this, we assume that its mass is given by a distribution which is the product of the Poisson probability that on average there exists a single subhalo more massive than it, by the probability of finding a subhalo at that mass. That is, its average value for a parent halo of mass M_h is given by

$$\langle M_{sh,2}(M_h) \rangle = \int_0^\infty M P_2(M, M_h) dM, \quad (12)$$

where $P_2(M, M_h)$ is the probability of the second most massive subhalo having mass M ,

$$P_2(M, M_h) = N(M|M_h) / \langle N(M|M_h) \rangle \exp(-\langle N(M|M_h) \rangle), \quad (13)$$

where $N(M|M_h)$ is the mass distribution function of the subhaloes, equation 1, and $\langle N(M|M_h) \rangle = \int_M^\infty N(M'|M_h) dM'$ is the average number of subhaloes more massive than M in a parent halo of mass M_h .

Since this mass threshold depends on the mass of the parent halo, in the end we obtain a condition on halo mass for it to be treated as a host to a rich cluster. Using our results, we find this to be $M_h > 2.85 \times 10^{14} h^{-1} M_\odot$.

If we now combine the distribution of all haloes more massive than this, together with their subhaloes, with the mass luminosity relation, we can obtain the luminosity function of galaxies in clusters. In fact, it is possible to make a rough prediction for its shape at the high end by comparing the luminosity of the brightest galaxy in the cluster (given by the solid line in figure 3, which represents the luminosity of the galaxy associated with the parent halo itself) with the total cluster luminosity. The average luminosity of the brightest galaxy can be estimated as $L_1 = \int_{L'}^\infty L \phi_{cl}(L) dL$, where $\phi_{cl}(L)$ is the cluster galaxy luminosity function, and L' is such that on average there is only one galaxy more luminous than L' , i.e. $\int_{L'}^\infty \phi_{cl}(L) dL = 1$ (see Yang, Mo & van den Bosch 2003). With $\phi_{cl}(L)$ normalized to the total cluster luminosity, L_{cl} , it is then possible to extract a relation between L_1 and L_{cl} , which will depend on the shape of the luminosity function. Taking our results from figure 3 that $L_1 \propto M^{0.28}$ and $L_{cl} \propto M^{0.9}$, it is possible to work out that, at the high end, $\phi_{cl}(L)$ is given by a power law with slope -4.21.

We show the results we obtain in figure 4, where we have excluded the first brightest galaxies (in effect, we are just accounting for the galaxies in subhaloes). For comparison, we include the 2dF global luminosity function of Norberg et al. (2002), as a dotted line. We also include the luminosity function for galaxies in clusters in the 2dF survey, as derived by de Propris et al. (2003), as the short dashed line. This is also given by a Schechter function, with parameters $\alpha = -1.28$ and $M_{b,J}^* - 5 \log_{10} h = -20.07$. Since the normalisation is not given, we adjusted this to have values in the same range as the ones we obtain. Also shown, as the long dashed line, is a double power law with the same low end slope, normalisation, and L_* as the ones used to plot the previous curve, but where the exponential cutoff has been replaced with a power law with the same slope as calculated above, namely -4.21. Our results show a good agreement to those obtained from 2dF, which is even better if a double power law is considered instead of a Schechter function.

3.2 Group luminosity function

Another way to check our results is to build the group luminosity function. This is the equivalent of the galaxy luminosity function, only applied to groups, and it is usually obtained from galaxy catalogues by building groups of gravitationally bound galaxies. In our case, we start with the same assumptions for groups described above, and use the

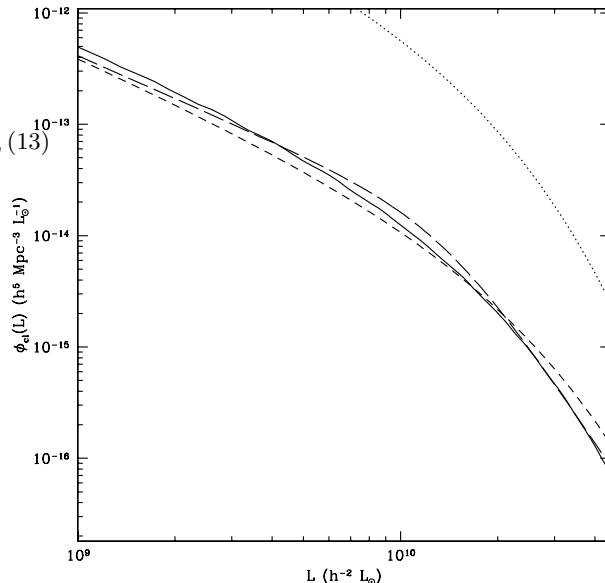


Figure 4. Luminosity function of galaxies in rich clusters. Shown as the solid line is our model, where we have excluded the first brightest galaxies, and where we take an effective threshold of $M_h > 2.85 \times 10^{14} h^{-1} M_\odot$ for the mass of a halo associated with a rich cluster. The dotted line shows the 2dF global luminosity function, the short dashed one the luminosity function of cluster galaxies also from 2dF, and the long dashed line is a double power with the same parameters as the previous function, but with $\phi_{cl}(L) dL \propto L^{-4.21}$ at high L .

halo mass function and the relation between halo mass and group luminosity given by equation (10) so that

$$\phi_g(L_g) dL_g = n_h(M(L_g)) \frac{dM}{dL_g} dL_g. \quad (14)$$

We show our result in figure 5, along with the AGS for the CfA survey (Moore, Frenk & White 1993) and the VSLF (Marinoni, Hudson & Giuricin 2002) group luminosity functions. While at higher luminosities our model would seem to underpredict the abundance of groups, it reproduces the slope of the AGS luminosity function quite well. This is undoubtedly related to the fact that our calculated group luminosity is somewhat lower than the observed values (see figure 3); an increase in group luminosity would cause a shift to the right of our curve, providing a better match for the observational results. It should also be noted that the inclusion of subsubhaloes would probably tend to slightly increase the values at the high end, relatively to the low end, since most of these subsubhaloes would be dark for low mass parent haloes, but be more massive and therefore contribute a small luminosity to the group total in the case of massive parent haloes. It is also curious to note that, since the fits shown in the figure are a double power law in the case of the AGS and a Schechter function in the case of the VSLF, our result would probably not be well fit by a Schechter type function.

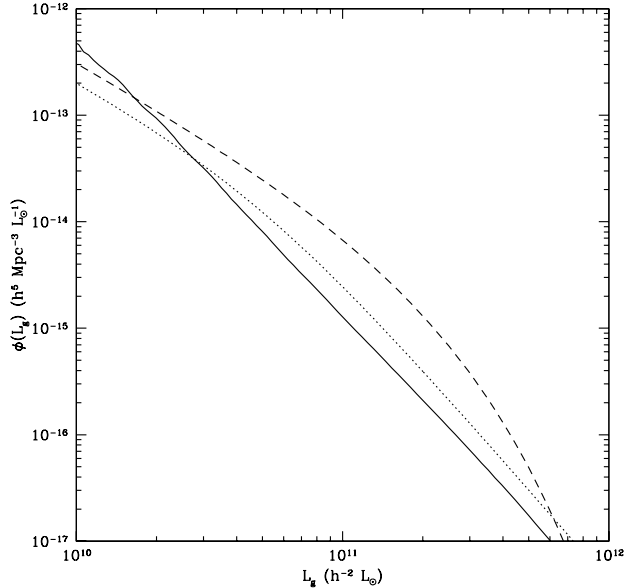


Figure 5. Group luminosity function. The solid line represents our result using equation (14); the two other lines are observational group luminosity functions extracted from galaxy catalogues, respectively the AGS (Moore, Frenk & White 1993) (dotted) and the VSLF (Marinoni, Hudson & Giuricin 2002) (dashed).

3.3 Multiplicity function

It is also possible to derive the multiplicity function, the number of groups/clusters as a function of their richness, by a process similar to the one used for the group luminosity function. Only in this case, we use the occupation number (plus one to account for the central galaxy, which is hosted by the parent halo itself) instead of the group luminosity. Then, using an expression similar to (14) with the subhalo number N_s put in place of the group luminosity L_g , we obtain the result shown in figure 6. The main point to take into account in this calculation is, as was referred in our above discussion of the occupation number, the need to introduce a minimum mass for the subhaloes. In the case of the figure shown, this was taken to be the mass equivalent to $M_B = -19.4$, but in general, and to compare to results from observational studies, this should be set to equal the minimum luminosity considered for objects in the observations. There is also a sharp upturn at $N = 1$ due to all the haloes that are massive enough to be considered to host a galaxy, but not enough so that they can have subhaloes with galaxies in them.

Figure 6 shows our results in comparison with some observational data. The points were taken from the analysis of Peacock & Smith (2000), while the two lines were constructed from the two group luminosity functions shown in figure 5, by using the luminosity functions of the two surveys to relate total group luminosity to the number of galaxies above the magnitude threshold. The galaxy numbers for both the points and the results derived from the group luminosity functions were then multiplied by a factor of 0.66 to take into account the difference in radius between friends

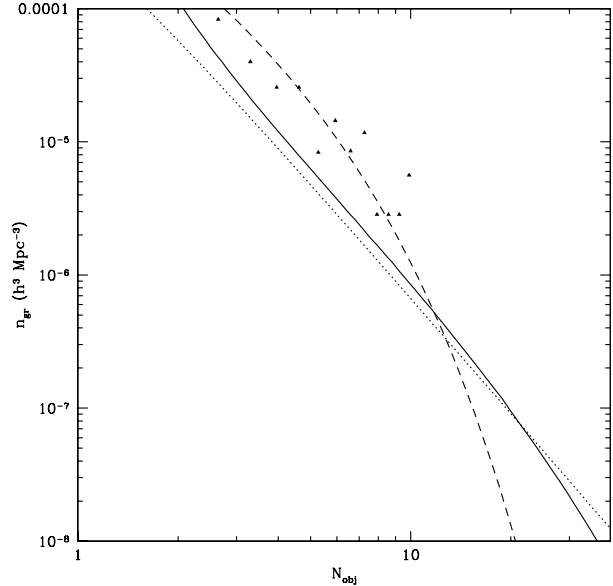


Figure 6. Multiplicity function derived from our model (solid line). This equates the number of galaxies in a group with the number of subhaloes in a halo, plus one to account for a central galaxy hosted by the halo itself. The minimum mass taken for the subhaloes was in this case the equivalent to a magnitude of $M_B = -19.4$. The different points are taken from the results derived by Peacock & Smith (2000) from the CfA survey, while the two additional lines are derived from the group luminosity functions shown in figure 5, where the magnitude limit for all of these is also $M_B = -19.4$.

of friends estimates and the usual definition of virial radius (see Kochanek et al. 2003).

Even though our results lie in the range between those estimated from the VSLF and the AGS group luminosity functions, they seem to be a bit lower than the observational points. Since the group abundance is directly related to the halo mass function, which is well known, this discrepancy must be caused by the occupation number. A slightly higher occupation number would shift our curve further to the right in the figure, bringing it into better agreement with the observational data. Incidentally, a higher occupation number would also bring our results for the group luminosity and the related group luminosity function into better agreement with observational values, so we believe that this is where the problem lies. Our calculated occupation numbers seem to be in fair agreement with expectations; however, they depend sensitively on our prescription for subhalo stripping, where we adopted a simplistic approach. A more detailed and correct model for this effect would most likely produce better results.

4 BIAS AND PROBABILITY FUNCTIONS

4.1 Mass, light and number densities

To derive a relation between the mass and light densities, we first need to obtain the mass function of haloes for regions of different density. In order to do this, we follow the

method outlined in Gottlöber et al. (2003). In essence, we treat the evolution of the matter distribution in a region as if it derived from a universe with the same cosmological parameters as that region, namely the same average mass density as is found in the local region. We start by labelling each region by its average mass density, ρ , to which corresponds a given value of the parameter $\Omega_m = \rho/\rho_c$, with ρ_c the critical density. Overdense regions will have $\Omega_m > \Omega_m$, where the barred quantities refer to the background universe, while voids will have the opposite.

The growth of perturbations will be affected by the different matter content, and this is reflected by changing the normalisation of the power spectrum according to the differential of the growth factor, $D(z)$, relative to the background:

$$\sigma_8 = \bar{\sigma}_8 \frac{\bar{D}(z_i)}{\bar{D}(0)} \frac{D(0)}{D(z_i)}, \quad (15)$$

where z_i is some initial redshift for which the fluctuations were equal in the background and in this region; in the present case, we use $z_i = 1000$. We also use the usual normalisation for the growth factor, $\bar{D}(0) = 1$. To obtain the mass function for a region with a particular average density, $n_\rho(m)dm$, we then apply this new normalisation to the Sheth Tormen mass function given in equation (6), changing the ρ_m term as appropriate, and also the value for δ_c , which has a small dependence on Ω_m (see for example Navarro, Frenk, & White 1997).

An important point is that this prescription does not utilise an explicit smoothing radius for the region considered. Instead, these are labelled by their average density. However, when regions of limited size are considered, there exists a maximum mass for objects in them, given by the total mass they contain. As the above prescription does not take this into account, we introduce an additional term to compensate for this effect. Therefore, after we obtain the mass function, we put in a further cutoff of the form $\exp[-\eta(m/m_\delta)^2]$, where m is the mass of the halo and $m_\delta = 4\pi(1+\delta_m)\bar{\rho}R^3/3$ the total mass in the region of radius R and average density $(1+\delta_m)\bar{\rho}$. The parameter η allows for some tuning of the actual cutoff, and in the present case we use $\eta = 1$. Once this cutoff is introduced, it becomes necessary to renormalise the mass function, so that it still gives the appropriate density when the mass of all haloes is calculated. Two examples of the resultant mass functions are shown in figure 7, with curves for regions with the average density, 1/10 of the average and 10 times the average, for two different radii.

Once the mass function for an over or under dense region has been determined, it is then simple to find the equivalent light density, ρ_L , by using the group luminosity correspondent to each halo, given by equation (10):

$$\rho_L = \int_0^\infty L_g(M)n_h(M)dM. \quad (16)$$

Our result is shown in figure 8. The results for $R = 8h^{-1}\text{Mpc}$ and $R = 4h^{-1}\text{Mpc}$ are quite similar, with the curve for $R = 1h^{-1}\text{Mpc}$ being different. This is due to the large suppression of high mass haloes, even at high overdensities, which can be seen in figure 7, which alters the proportion of luminous to non-luminous haloes in favour of the latter. The curve we obtain for $R = 8h^{-1}\text{Mpc}$ is similar to the one in Mo et al. (2003), where the authors have used a

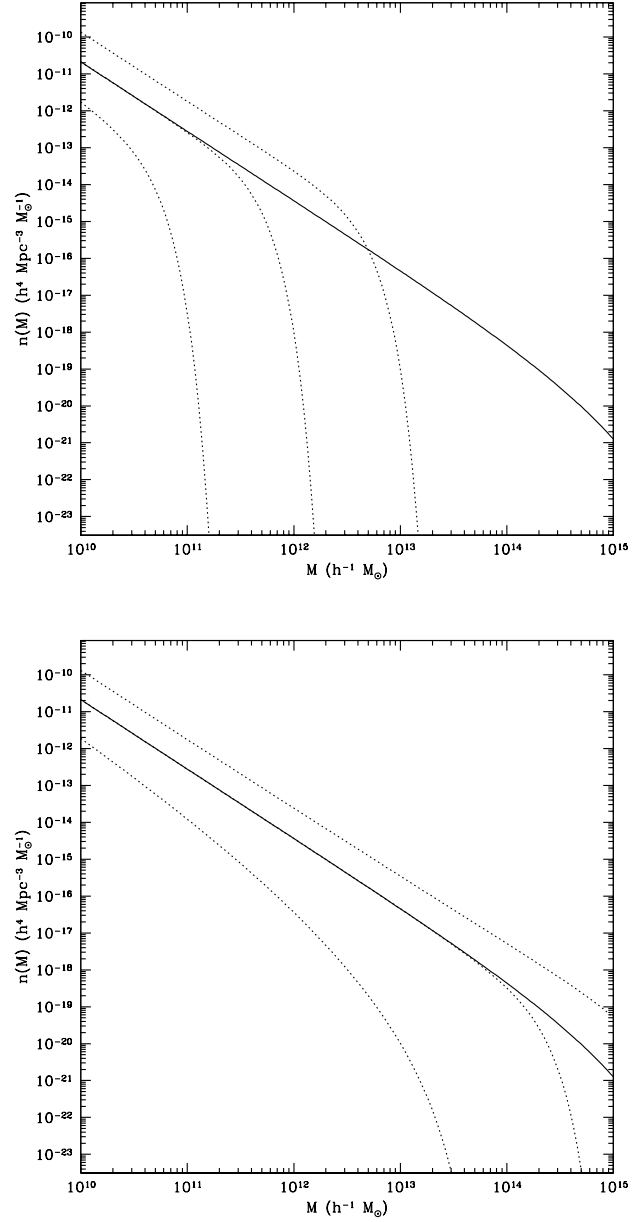


Figure 7. Mass functions for regions of different size and average density, calculated according to the prescription described in the text. The upper panel shows the results for $R = 1h^{-1}\text{Mpc}$, the lower for $8h^{-1}\text{Mpc}$. In both, the solid curves are the background Sheth Tormen mass function, the dotted ones are for densities of (top to bottom): $10\bar{\rho}$, $1\bar{\rho}$, $0.1\bar{\rho}$.

mass function for different densities derived from simulation results, to which they then apply the conditional luminosity function. The agreement is quite good with our result, even though we used a theoretical model for the mass function instead of taking it from simulations.

The most striking feature in these results is the sharp decline in the light density in underdense regions. This would mean that the majority of void regions would be quite dark. Another important feature is that for the most part $\delta_L < \delta_m$, and the slope of the curve at high density is lower than 1.

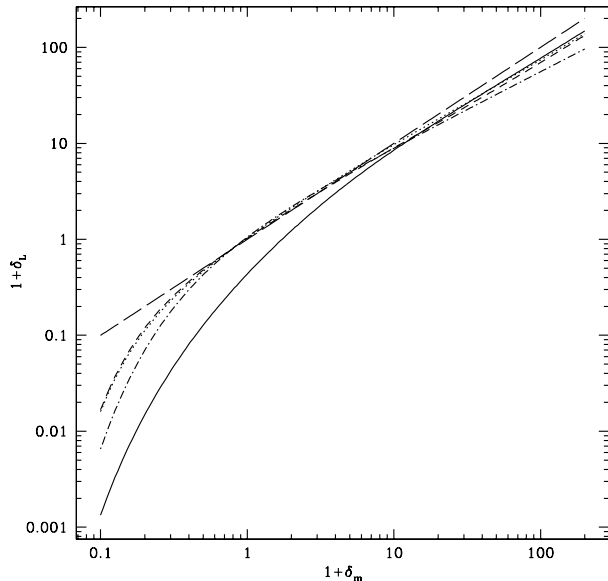


Figure 8. Light density, δ_L , as a function of mass density, δ_m . Both axis are scaled to the average background values, so regions with $1 + \delta_m > 1$ are overdense and regions with $1 + \delta_m < 1$ are underdense. The different lines are for different smoothing lengths: $R = 1h^{-1}\text{Mpc}$ (solid), $R = 4h^{-1}\text{Mpc}$ (dotted), $R = 8h^{-1}\text{Mpc}$ (short dashed). The long dashed line marks $\delta_L = \delta_m$. The dashed dotted line is the fit to the same relation mentioned in Mo et al. (2003).

The overall results (e.g., darkness of the voids) are similar to those from hydrodynamical simulations by Ostriker et al. (2003). But the simulation results show a sharper cutoff at low mass density and light being more overdense than mass for overdense regions and a slope to the relation greater than one. This difference is the more remarkable since the results of Mo et al. (2003) are similar to ours while using a mass function derived from simulations, which otherwise might be considered the most likely origin of the discrepancy. However, previous results shown in Bahcall et al. (2000), also taken from simulations, seem to be more in line with what we obtain in the present work. These authors in fact give an explanation for the apparent increasing antibias at higher mass densities: at low redshift these regions usually consist of rich clusters and superclusters, whose stellar population tends to be old. Therefore, young blue stars tend to be rare and consequently the total luminosity in the blue band is lower than what could be expected from their high mass, giving rise to the slight antibias. It is therefore likely that this difference is coming from the way in which light is being counted in the two methods. In fact, one problem with the approach presented here is that it breaks down for haloes whose size is comparable to the size of the region being considered, since it is then possible for a sphere to encompass an outer region of the halo where a subhalo and a galaxy lie, and which is therefore luminous but is considered dark in our prescription. Since Ostriker et al. (2003) study the light distribution directly, they would account for the light present in such a situation. This effect would be the more noticeable for high density regions and small smoothing lengths,

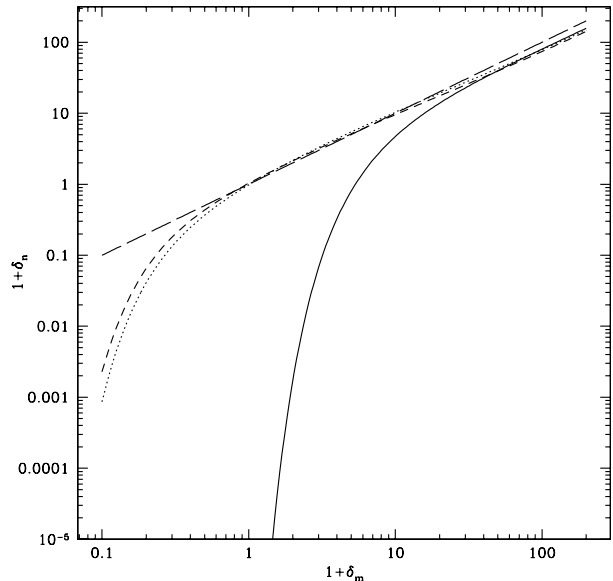


Figure 9. Same as figure 8 (curves have same label), but for δ_g , the overdensity in the number of galaxies. The minimum threshold was set at halo mass corresponding to a luminosity $L = L_*$. The average number of galaxies brighter than this limit in the background, taken from the luminosity function, is $3.22 \times 10^{-3} h^3 \text{Mpc}^{-3}$.

and in fact the results presented in Ostriker et al. (2003) are for $R = 1h^{-1}\text{Mpc}$, while the major discrepancy with our results occurs for high density regions. The differences in underdense regions can most likely be explained by limited resolution effects.

We also derive results for the bias between galaxy numbers and the dark matter mass density. The procedure is similar to the adopted for the luminosity, replacing the $L_g(M)$ term in (16) with the number of galaxies in a halo of mass M , i.e. $(1 + N_s(M, m_{\min}))$, corresponding to the central galaxy hosted in the halo and the ones present in the subhaloes. The integral should also be taken with a lower limit at m_{\min} . The minimum mass m_{\min} corresponds to the intended cutoff in mass (or luminosity) for the galaxies to be counted. Our result is shown in figure 9. The general behaviour is similar to what was observed for the luminosity. According to these results, the galaxy number distribution would be a fairly unbiased tracer of the underlying dark matter mass in moderately overdense regions, as would be expected from previous studies (e.g., Benson et al. 2001), but the galaxy numbers severely underestimate the mass density in underdense regions and consequently assuming a direct proportionality can lead to a significant underestimate of Ω_m (Ostriker et al. 2003).

4.2 Distribution functions

Using the relations obtained in the previous section, it is also possible to derive a distribution function for the light density (i.e., the probability of a region of a given size having a certain average light density, the light PDF). To make this calculation, besides the relation between the mass and

light densities it is also necessary to have the mass distribution function. In general, this can be obtained from dark matter simulations. For the present work, we will use the approximation of treating this as a lognormal, with an appropriate dispersion dependent on the radius being considered and on the cosmological model. It has been known for some time that such a function is a fair approximation to the real mass distribution function, at least in regions of moderate over/under density (Coles & Jones 1991; see also Kayo, Taruya & Suto 2001 and Ostriker et al. 2003 for some recent analysis on this subject).

The form of the lognormal is given by:

$$f(y) = \frac{1}{\sqrt{2\pi\omega^2}} \frac{1}{y} \exp\left(-\frac{[\ln(y) + \omega^2/2]^2}{2\omega^2}\right), \quad (17)$$

where $y = 1 + \delta_m$ and the parameter of the distribution ω is related to the smoothing length and the variance σ of the mass density field by

$$\omega_R^2 = \ln(1 + \sigma_R^2). \quad (18)$$

Here, the variance σ refers to the full, nonlinear spectrum. To relate this to the linear variance, and hence to the cosmological parameters we use, we follow the prescription outlined in Peacock & Dodds (1996) to convert the linear power spectrum of mass density fluctuations to the nonlinear one.

To obtain the light distribution function, we then follow a similar procedure to the one used for the group luminosity function:

$$g(j)dj = f(\rho) \frac{d\rho}{dj} dj, \quad (19)$$

where we use the notation $j = 1 + \delta_L$ and $g(j)dj$ is the light distribution function. Such a transformation assumes a monotonic dependence of the light density with the mass density, which is true of our results (see figure 8). Another important point to note is that this function is not normalised to one, since some of the underdense areas (in terms of mass) are non luminous (see discussion in Ostriker et al. 2003). Our result is shown in figure 10. The general behaviour is what would be expected: the distribution functions are similar in the overdense regions, but in general differ substantially in the underdense regions, with the light distribution function in general not well fit by a lognormal.

4.3 Void probability function

The void probability function is defined as the probability of having no galaxies in a sphere of radius R . Although voids are a particularly striking feature in the galaxy distribution, their study is impaired by their large size and low number of galaxies present in the observational case, and by resolution difficulties when the study is conducted through numerical simulations including galaxy formation. Due to these factors, in the past studies of the voids in the galaxy distribution have been relatively few (see, for example, Vogeley et al. 1994 for observational results, and Mathis & White 2002 and Benson et al. 2003b for comparisons with theoretical results; there are also two recent papers with observational results for the 2dF survey, Hoyle & Vogeley 2003; Croton et al. 2004). Despite this, the void probability is a powerful tool in the analysis of a particular model of galaxy formation, since it probes a highly non-linear regime and is

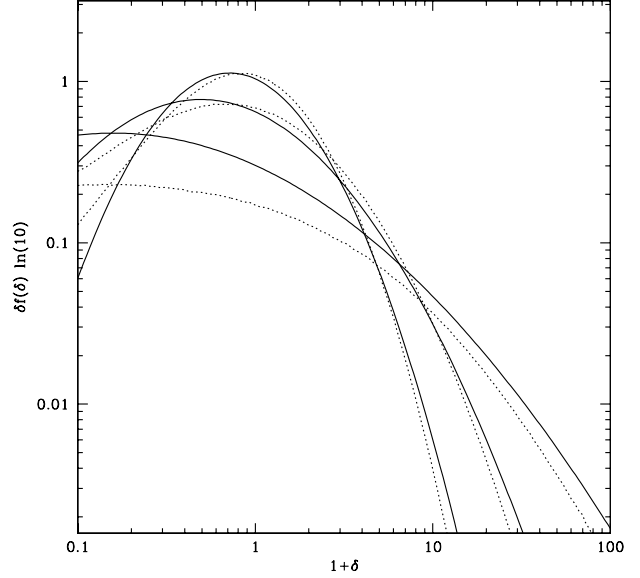


Figure 10. Distribution functions for mass (solid) and light (dotted) density. The three curves correspond to the three smoothing lengths used previously: 1, 4 and $8 h^3 \text{Mpc}^{-3}$, with peak height increasing with radius. The x-axis represents either mass or light overdensity, depending on the curve. The distribution functions are shown scaled appropriately for logarithmic binning.

not derivable from the low order correlation functions (in fact, it depends on all of the N-point correlation functions).

In the case of our model, we can determine the void probability function by combining the dark matter distribution function with the expected galaxy number in regions of a given size. Thus the number density of galaxies in a region of radius R and average density ρ , given a minimum mass cutoff of M_{min} will be:

$$n_R(\rho) = \int_{M_{min}}^{\infty} (1 + N_s(M)) n_{\rho,R}(M) dM, \quad (20)$$

where $n_{\rho,R}(M)$ is the mass function in a region of size R and average density ρ (see discussion in section 4.1, also figure 7), and $N_s(M)$ is the number of subhaloes in a halo of mass M . The number is then obtained by multiplying this density by the region volume, $V = 4\pi R^3/3$. We treat this as the average number and assume a Poissonian distribution around this average (see Kravtsov et al. (2003), where the authors have shown that the full HOD is consistent with a Poisson distribution for large host masses), so that the probability of having 0 galaxies will in fact be $\exp(-V n_R(\rho))$. Finally, the void probability function is obtained by integrating this number times the probability of a region having this given density,

$$P_0(R) = \int_0^{\infty} \exp(-V n_R(\rho)) f_R(\rho) d\rho, \quad (21)$$

where $f_R(\rho)$ is the probability of a region of radius R having density ρ , which we take to be given by the lognormal distribution, equation (17).

Our results are shown in figure 11, where we also show observational results for the 2dF survey taken from

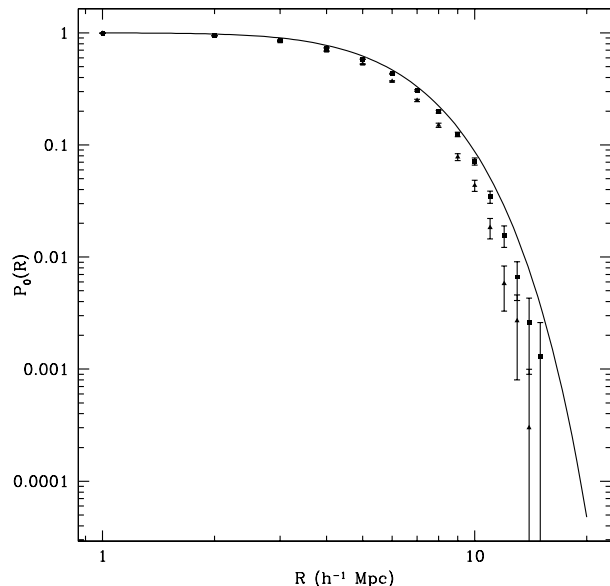


Figure 11. Void probability function. The solid line represents our concordance model. The points are from the combined CfA-1 and CfA-2 surveys (taken from Benson et al. 2003b, original results in Vogeley et al. 1994). The observations are for a magnitude limit of $M_B - 5\log h \leq -19.5$, and the corresponding mass was taken as the minimum mass limit in the calculations (equation 20).

Hoyle & Vogeley (2003), for both the NGP and the SGP. At low radius, we obtain a good agreement, but our model would seem to overestimate the probability at large radii. Our results are also in better agreement with the values observed for the SGP than for the NGP, since Hoyle & Vogeley (2003) have found the former to be somewhat emptier than the latter. There is an additional effect that may go some way towards explaining this discrepancy, and that is corrections for the peculiar velocity distortions. These would lead, especially for the larger radii, to the inclusion in voids of galaxies that are not actually there, due to the smearing effect caused by peculiar velocities to the positions of galaxies in redshift space. This effect is not symmetric, since the voids are areas of low mass density and therefore the galaxies in them will have low velocity dispersions. The final result of this would be to overpopulate the voids and consequently to underestimate the measured void probability function, which would lead to a better agreement with our results.

5 CONCLUSION

In this paper we present a new model for relating halo mass to hosted galaxy luminosity, based on the dark matter substructure. We feel this is a potentially powerful way to approach this problem, since it is based on two main inputs, the halo/subhalo distribution and the galaxy luminosity function, which can be well tested and adjusted to results from simulations and surveys, respectively. Additionally, the model requires only one further assumption: that there is a one to one and monotonic relation between imbedded halo/subhalo mass and galaxy luminosity. This is more

explicit but in general agreement with the general assumptions made in past studies and general assumptions made in similar work of the same subject. It is a much less restrictive assumption than the still sometimes utilized ansatz of a linear “bias” between galaxy numbers and dark matter density.

We have shown how, starting with a prescription we describe for the subhalo mass distribution in a parent halo, it is possible to obtain a relation for the luminosity of a hosted galaxy, as well as the group luminosity when the system of a halo and its subhaloes is identified with groups or clusters of galaxies. The subsequent results appear to match well with general assumptions of the behaviour of such a relation, as well as results for the mass and luminosity at scales of dwarf galaxies, L^* galaxies and massive clusters.

From this model, it is then possible to derive many quantities that can be directly compared with further simulation or observational results. We have shown four examples of this, namely the occupation number, the luminosity function of cluster galaxies, the group luminosity function and the multiplicity function. Using further assumptions on how to calculate the mass function for regions of different average densities, and on the shape of the dark matter distribution function, we have also obtained a relation between mass and light and number densities for different smoothing lengths close to what is expected from previous bias studies. We also obtain the distribution function of light density, and the void probability function. The latter is a powerful additional test, since it probes a highly non-linear regime, and our results seem to match well with previous observations.

The major difficulty with the model as it is presented here has to do with the identification of the mass of the subhaloes. To be able to apply the monotonic correspondence to the galaxy luminosity, the original mass in the subhaloes has to be taken into account. However, the mass distribution we use is measured for the stripped mass of the subhaloes in their parent halo. In order to build the model, we took the approximation of taking an average for the stripping factor. This is, however, a not wholly satisfactory approach, since in general the stripping history will be highly variable from subhalo to subhalo, and also in different parent haloes. Ideally, we would like to use the maximum circular velocity instead of the mass to identify the subhaloes, since this quantity should be less sensitive to stripping for the relatively massive subhaloes which can host galaxies. Unfortunately, we do not have at present a good distribution for subhalo abundance as a function of the maximum circular velocity, although we wish to study this further in the future. Nonetheless, although somewhat crude, the approximation we took is reasonable, and since this work deals with statistical averages for most of the quantities, it does fit somewhat well into the whole structure. More importantly, the results we obtain seem for the most part to match well with observational data. In future work which takes into account the statistical variation, however, this factor will certainly be of great importance.

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